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IYPT

International Young Physicists' Tournament

Hula hoop dance without a dancer

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3. Ring on the Rod

A washer on a vertical steel rod may start spinning instead of simply sliding down. Study the motion of the washer and investigate what determines the terminal velocity.

One ring, one rod and the gravity force is all that is needed to observe the phenomenon.

With the apparent lack of other forces, what causes this unexpected effect?

Sometimes we have constraints at work.
They specify the type of interactions and may provide the essence of the problem.

For example, one well-known effect is when a ball rolled on the inside of a rough vertical cylinder does not descend constantly but recurrently goes up and down in the case of **no slip rolling**.

(This may be rigorously proved in the idealised case and could be observed for a limited amount of time.)

The basic qualitative explanation of the washer's behavior is obvious – it rolls around the rod and due to the centripetal force there is a normal reaction and hence a friction force countering the gravity. As long as the angular velocity is big enough to assure that static friction is bigger than the weight, the washer shall keep its height constant.

$$\mu \frac{m\, v^2}{d} \;(\doteq \mu\, m\, \omega^2\, d)\; \geq m\, g$$

Also, when the angular velocity is big enough the washer shall be practically in a horizontal position and this is what we observe in the initial phase of our experiments.

What about ***no slip rolling***?

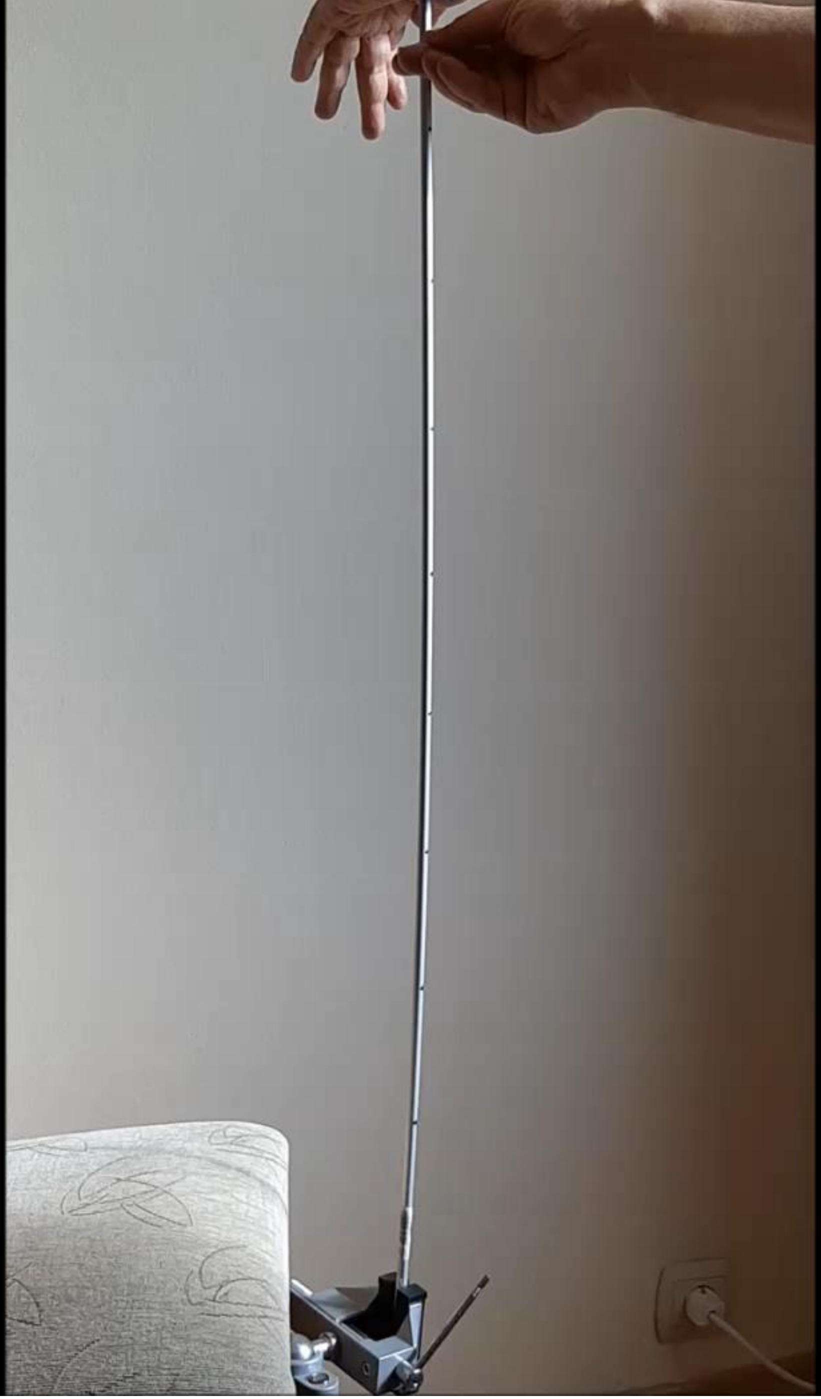
In some experiments one can see the washer moving up for a while but this could not be taken as an ultimate proof that we have *no slip rolling* during this phase, esp. after a big initial spin.

An experimental measuring of the two angular velocities (rotation of the washer around its center vs the rotation of the center of the washer around the rod) would be required in order to verify this hypothesis.

The coefficients of static and dynamic friction could be determined e.g. by putting the ring on a horizontal rod and slowly increasing the inclination angle till sliding begins.

Finding out the coefficient of rotational friction may be trickier but one may try putting the ring on a horizontal rod and making it oscillate without tilt in the plane perpendicular to the rod. The rate of slowing down the oscillations could be used to provide the needed estimation [c.f. R. Cross “Coulom law for rolling friction” Am J Phys **84** (2016) p.221].

As angular velocity slows down (mostly due to the rotational friction) the washer tilts and after overcoming the static friction it starts descending. In certain cases this is the end of the story as the ring falls down at the base of the rod.



The interesting point is that sometimes one may observe here a transition to a **different regime characterised by a great tilt** (as if having two points of contact).

Both the tilt and the velocity of descent seem to be constant during this phase.

This motion deserves meticulous investigation, say, by using two high-speed cameras, one of it shooting from the top and combining the data.

There is a number of points to be clarified:

- Is the downward velocity really constant? For how long?
- Do we have rolling without slipping here?
- Do we have two points of contact or just one? → Maybe one
- Is the uppermost tip of the washer’s hole a point of contact? → Maybe no
- What is the line followed by the point of contact? A helix?
- …

Theoretical explanation…

While the rest of the regimes could be more or less trivially explained, the last one presents a challenging task.

After specifying the behavior of the kinematic parameters one may ponder why and when this regime is possible.

For example, one may consider the Coriolis force caused by the presence of downwards linear velocity and horizontal component of the angular velocity; how angular momentum changes with time etc...

Or trying the model of a no slip roll along a helix line…

More teasers:

- *Could the washer start spinning by itself?*
- *What is the dependence on the mass of the washer?*
- *Investigate the effect of the ring geometry. (There are some hints that the optimal size of the ring's hole should be about twice the rod's diameter…)*
- *One may see in YouTube demonstrations with a threaded rod. This may be a deviation from the definition of the problem but nevertheless it may be interesting to discuss what shall change in this case.*
- *What happens if we have a rod with a (small) constant tilt or a wobbling one?*
- *Can you specify the conditions for occurrence of different regimes? Can you show some sort of a 'phase' diagram in the space of relevant parameters?*

Literature (just bits of information)

- R. Cross, *Rotating ring on a vertical rod*, Phys. Educ. **56** (2021) 023003
- R. Cross, *Effect of friction on a hula hoop*, Phys. Educ. **56** (2021) 033001
- H. Crane, *Chattering, the chatterring, and the hula hoop*, The Physics Teacher **30** (1992) 306;
- Some more videos: http://www2.eng.cam.ac.uk/~hemh1/movies.htm
- A curious calculation (in Russian) for the case when the slip velocity is proportional to the rolling velocity: https://iypt.ru/wp-content/uploads/2021/09/%D0%92%D1%8B%D1%81%D1%88%D0%B0%D1%8F-%D0%BF%D1%80%D0%BE%D0%B1%D0%B0-%D1%80%D0%B0%D0%B7%D0%B1%D0%BE%D1%80-2018.pdf

Giving initial spin